Derivation of Wave Equation from Maxwell's Equations

Charge Conservation: $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

Faraday's Law: $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times \nabla \times E = \nabla \times \left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times B)$$

Ampere's Law: $-\frac{\partial}{\partial t}(\nabla \times B) = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

vector identity: $\nabla \times (\nabla \times E) = -\nabla^2 E$

wave equation: $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

similarly: $\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$

$$\mu_0 \epsilon_0 = 1/c^2$$