Eigenmode decomposition of arbitrary field configurations

Eigenmode decomposition exploits Meep's interconnectivity with the MPB mode solver to express an arbitrary time-harmonic field configuration as a superposition of the normal harmonic modes of your structure.

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Theoretical background[1]

Consider a waveguide structure of infinite extent in the x direction with constant cross section in the transverse $[\vec{\rho} = (y, z)]$ directions. For any given angular frequency ω we may solve the time-harmonic Maxwell equations to obtain the *normal modes* of the structure---an infinite set of vector-valued functions of the transverse coordinates $\{\mathbf{E}_n^{\pm}(\vec{\rho}), \mathbf{H}_n^{\pm}(\vec{\rho})\}$, with

associated propagation constants $\{\beta_n\}$, that furnish a complete expansion basis for time-harmonic electromagnetic fields at frequency ω. That is, given any arbitrary frequency-ω field configuration of the form

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{-i\omega t}$$

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})e^{-i\omega t}$$

we have the *exact* expansions

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(x,\vec{\rho}) = \sum_n \left\{\alpha_n^+ \mathbf{E}_n^+(\vec{\rho})e^{+i\beta_n x} + \alpha_n^- \mathbf{E}_n^-(\vec{\rho})e^{-i\beta_n x}\right\} \tag{1}$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}(x,\vec{\rho}) = \sum_n \left\{\alpha_n^+ \mathbf{H}_n^+(\vec{\rho})e^{+i\beta_n x} + \alpha_n^- \mathbf{H}_n^-(\vec{\rho})e^{-i\beta_n x}\right\} \tag{2}$$

where (as discussed further below) the expansion coefficients $\{\alpha_n^\pm\}$ may be extracted from knowledge of the time-harmonic fields $\mathbf{E},\mathbf{H}$ on any cross-sectional surface S transverse to the waveguide.

The idea of mode expansion in Meep is to compute the $\{\alpha_n^\pm\}$ coefficients above for any *arbitrary* time-harmonic field distribution resulting from a Meep calculation. In calculations of this sort,

- the $\{\mathbf{E},\mathbf{H}\}$ fields on the RHS of equations (1a,b) above will be frequency-domain fields stored in a `dft_flux` object in a Meep run, where you will have arranged this `dft_flux` object to live on a cross-sectional surface S transverse to the waveguide;
- the $\{\mathbf{E}_n^\pm,\mathbf{H}_n^\pm\}$ eigenmodes and $\{\beta_n\}$ propagation constants are computed automatically under the hood by MPB as normal modes of an infinitely extended waveguide with the same cross-sectional material distribution that your structure has on the transverse slice S, and
- the $\alpha_n^\pm$ coefficients for as many bands as you like are computed by calling `get_eigenmode_coefficients()`, as discussed below.

Main function prototype

The highest-level interface to the mode-expansion implementation in Meep is the libmeep function `meep::fields::get_eigenmode_coefficients`, callable from C++ or python. This routine makes use of several lower-level libmeep functions that you may also find useful; these are documented below and their use is illustrated in the tutorial that follows.

The C++ prototype for the top-level routine is

```
std::vector<cdouble>
 fields::get_eigenmode_coefficients(dft_flux *flux,
                                    direction d,
                                    const volume &where,
                                    std::vector<int> ban
                                    kpoint_func k_func=0
                                    void *user_data=0);
```

where

- `flux` is a `dft_flux` object pre-populated with frequency-domain field data resulting from a time-domain Meep calculation you have run to tabulate fields on a cross-sectional slice perpendicular to your waveguide
- `d` is the direction of power flow in the waveguide
- `where` is a `volume` describing the cross-sectional surface S
- `bands` is an array of integers that you populate with the indices of the modes for which you want expansion coefficients
- `user_func` is an *optional* function you supply to provide initial estimates of the wavevector of a mode with given frequency and band index; its prototype is

```
vec (*kpoint_func)(void user_data, double freq, int ban
```

which returns a `vec` giving your best guess for the wavevector of the `band` th mode at frequency `freq` .

The return value of `get_mode_coefficients` is an array of type `cdouble` (short for `std::complex<double>`), of length `2 * num_freqs * num_bands` , where `num_freqs` is the number of frequencies stored in your `flux` object (equal to `flux->Nfreq`) and `num_bands` is the length of your `bands` input array. The expansion coefficients $\{\alpha^+, \alpha^-\}$ for the mode with frequency `nf` and band index `nb` are stored sequentially starting at slot `2*nb*num_freqs + nf` of this array:

```
std::vector<cdouble> coeffs=f.get_eigenmode_coefficient
fields::get_eigenmode_coefficients(dft_flux *flux,
                                   direction d,
                                   const volume &where,
                                   std::vector<int> ban
                                   kpoint_func k_func=0
                                   void *user_data=0);

int num_bands = bands.size();
int num_freqs = Flux->Nfreq;
for(int nb=0; nb<num_bands; nb++)
 for(int nf=0; nf<num_freqs++; nf++)
  {
    // get coefficients of forward- and backward-travel
    // waves in eigenmode bands[nb] at frequency #nf
    cdouble AlphaPlus  = coeffs[2*nb*num_freqs + nf + 0
    cdouble AlphaMinus = coeffs[2*nb*num_freqs + nf + 1
    ...
```

Sample application: tapering between waveguides

As a demonstration of mode expansion, we'll consider the problem of *tapering* between waveguides of different sizes. More specifically, we'll suppose we have incoming power, carried by a single mode (typically the fundamental mode) of a first waveguide (waveguide A) that we wish to route into a single mode (typically the same mode) of a second, larger, waveguide (waveguide B), losing as little power as possible to reflections or inter-mode scattering in the process. Simply jamming the ends of the two waveguides together will result in significant losses due to the abrupt "impedance" mismatch at the interface, so instead we will consider gradually morphing ("tapering") the waveguide cross section from that of waveguide A to that of waveguide B over a finite length *L*---with a taper profile of smoothness index p---and study the dependence of the mode-to-mode power transfer on L and p.

The calculations described below are implemented by a python code called `wvg-taper.py`, which we will dissect as we proceed through the example. A C++ version of the same calculation is `wvg-taper.cpp`.

First calculation: 2D geometry

As a first example of relatively modest computational cost, we'll consider a 2D (z-invariant) problem in which the smaller and larger waveguides are simply finite-thickness slabs of dielectric material suspended in vacuum. More specifically, power travels in the *x* direction with the fields confined by the dielectric in the *y* direction; the smaller and larger waveguides have thicknesses w_A and $w_B \geq w_A$ and are connected by a taper region of length L, so the slab thickness as a function of x reads

$$w(x) = \begin{cases} w_A, & x < -\frac{L}{2} \\ T_p(x), & x \in \left[-\frac{L}{2}, +\frac{L}{2}\right] \\ w_B, & x > +\frac{L}{2} \end{cases} \tag{3}$$

where the taper function $T_p(x)$ is a C^p function, i.e. p is the index of its first discontinuous derivative. For the cases $p = \{0, 1, 2\}$, the taper functions are

$$T_p(x) = \begin{cases} w_0 + \Delta\left(\frac{x}{L}\right), & p = 0 \\ w_0 + \Delta\left[\frac{3}{2}\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^3\right], & p = 1, \\ w_0 + \Delta\left[\frac{15}{8}\left(\frac{x}{L}\right) - 5\left(\frac{x}{L}\right)^3 + 6\left(\frac{x}{L}\right)^5\right], & p = 2 \end{cases}$$

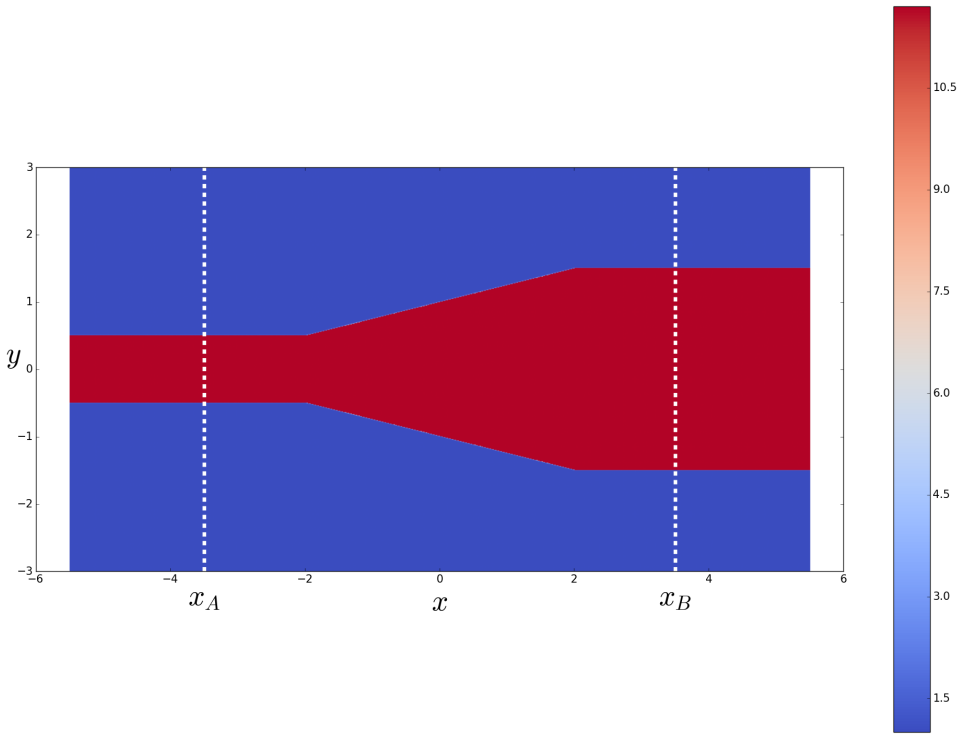
where

$$w_0 \equiv \frac{w_A + w_B}{2}, \qquad \Delta = w_B - w_A$$

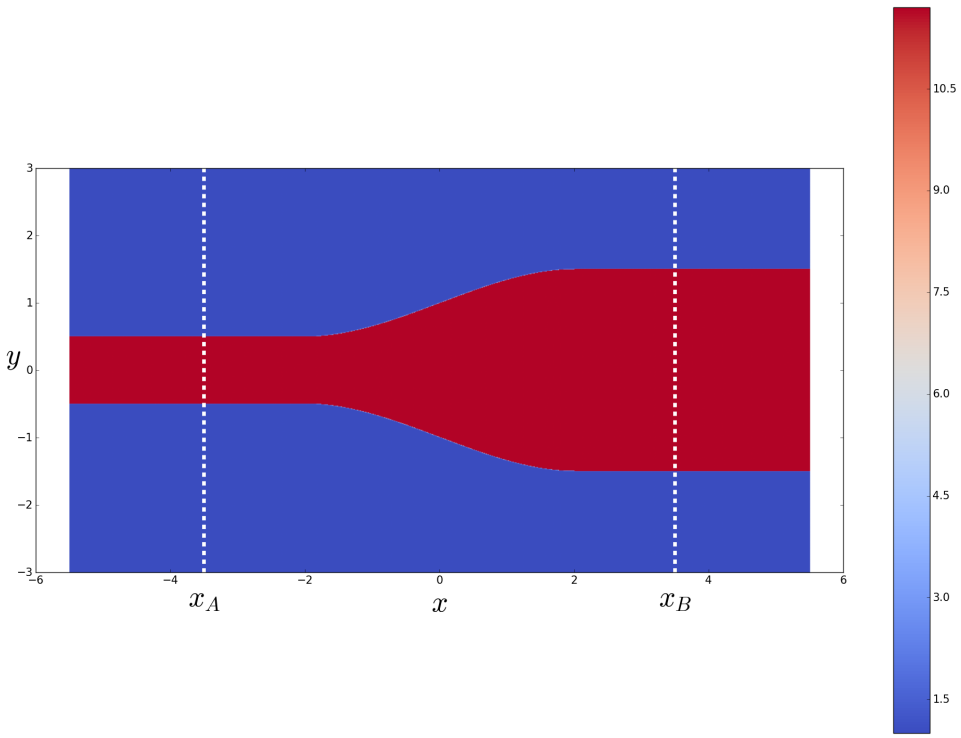
are the average and difference of the smaller and larger waveguide thicknesses.

Here are pictures of the $p = 0, 1, 2$ taper geometries for the case of a taper of length $L = 4$ between waveguides of thickness $w_A = 1$ and $w_B = 3$. (See below for the python code that produces these plots.)

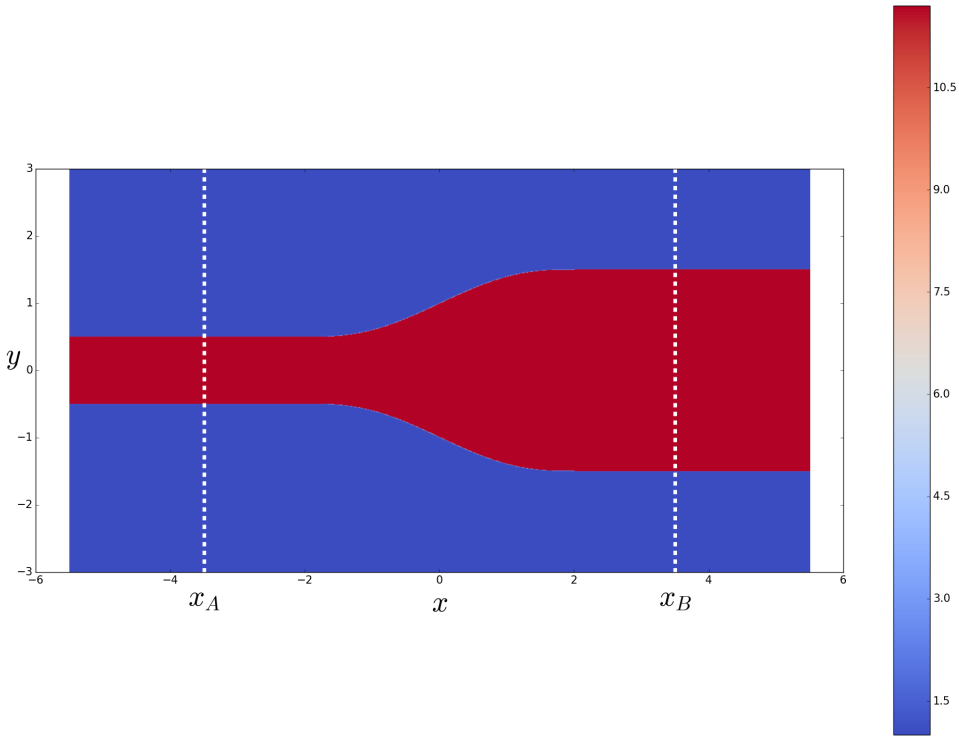
***p*=0 Taper**


3
2
1
0
−1
−2
−3
y
−6
−4
−2
0
2
4
6
x_A
x
x_B
10.5
9.0
7.5
6.0
4.5
3.0
1.5

p=1 Taper


3
2
1
0
−1
−2
−3
y
−6
−4
−2
0
2
4
6
x_A
x
x_B
10.5
9.0
7.5
6.0
4.5
3.0
1.5

p=2 Taper



In these figures, the dashed lines at $x = x_{A,B}$ indicate the locations of cross-sectional planes that we will use in our calculation: the plane at $x = x_A$ is where we will place an eigenmode source in our Meep calculation to describe incoming power entering from the smaller waveguide, while the plane at $x = x_B$ is where we will tabulate the Fourier-domain fields in our Meep calculation and determine their overlaps with the eigenmodes of the larger waveguide to compute mode-expansion coefficients.

User-defined material function

Because the material geometries we will be studying here are too complicated to be described as assemblies of the usual geometric primitives like blocks and cylinders, we will instead write our own user-defined material function, which inputs the coordinates of a point in space and fills in a medium structure for the material properties at that point. Actually, since the material geometry in this case involves a spatially-varying but otherwise simple (isotropic, linear, lossless) dielectric function, we can get away with the slightly simpler user-defined epsilon function, for which we need only furnish a function of position that returns a scalar relative permittivity. This is implemented by the `my_eps_func()` routine in `wvg-taper.py`; note that it invokes a subroutine `w_func` that evaluates equation (3) above to compute the x-dependent waveguide width $w(x)$.

```
##################################################
# x-dependent width of waveguide
##################################################
def w_func(x, L, p, wA, wB):
  if L==0:
    return wA if x<0 else wB
  x0=x/L
  if (x0 < -0.5):
    return wA;
  elif (x0 > +0.5):
    return wB;
  elif p==2:
    return 0.5*(wA+wB) + (wB-wA)*x0*(15.0 + x0*x0*(-40.0
  elif p==1:
    return 0.5*(wA+wB) + (wB-wA)*x0*(1.5 - 2.0*x0*x0);
  else: # default t p==0, simple linear taper
    return 0.5*(wA+wB) + (wB-wA)*x0;

##################################################
# user-defined function for position-dependent material
##################################################
def my_eps_func(loc, L, p, wA, wB, eps_out, eps_in):

    if ( abs(loc.y) > 0.5*w_func(loc.x, L, p, wA, wB) ):
     return eps_out;    # outside waveguide
    else:
     return eps_in;     # inside waveguide
```

We can pass `my_eps_func` as the value of the `epsilon_func` keyword argument to the `Simulation` class constructor; however, because this expects a function of just a single argument (the spatial point), we use a `lambda` construction to package the remaining arguments, i.e. something like

```
eps_func = lambda loc: my_eps_func(loc, L, p, wA, wB,
                                   eps_ambient, eps_wave

sim=mp.Simulation( cell_size=mp.Vector3(2*LX, 2*LY),
                   resolution=resolution,
                   boundary_layers=[mp.PML(DPML)],
                   epsilon_func = eps_func
                 )
```

The `wvg-taper.py` code defines a class called `wvg-taper` that accepts keyword arguments for various geometric parameters and instantiates a `Simulation` object as in the code snippet above. For example, here's how we made the pictures of the structures shown above: a couple of examples involving waveguides and tapers of various geometries:

```
>>> execfile("wvg-taper.py");
>>> wt=wvg_taper(wA=1, wB=3, L=4, p=0); wt.plot_eps();
>>> wt=wvg_taper(wA=1, wB=3, L=4, p=1); wt.plot_eps();
>>> wt=wvg_taper(wA=1, wB=3, L=4, p=2); wt.plot_eps();
```

The `plot_eps()` class method that produces these plots just calls `Simulation.get_array` to get a `numpy` array of ε values at the grid points, then plots it using the `imshow` routine in matplotlib:

```
    def plot_eps(self):

     eps=self.sim.get_array(center    = mp.Vector3(0,0),
                            size      = self.sim.cell_si
                            component = mp.Dielectric)
     plt.figure()
     plt.imshow(eps.transpose())
     plt.show(block=False)
```

Visualizing eigenmode profiles

Next, before doing any timestepping let's calculate and plot the field profiles of some waveguide modes, for both the smaller and larger waveguides. This calculation is done by the `plot_modes` function in the `wvg_taper` class; you can look at the full Python code to see how it's done in full detail, but here is a synopsys:

- For the lowest-frequency ($n = 1$) eigenmode of the smaller waveguide, and for the first several eigenmodes of the larger waveguide, we call the `meep::fields::get_eigenmode` routine in libmeep. This routine inputs a frequency (`fcen`), an integer (`nb`), and a `meep::volume` specifying a cross-sectional slice through our geometry, then invokes MPB to determine the `nb` th eigenmode at frequency `fcen` for an *infinitely extended* waveguide with constant cross section matching that of our slice. For example, to compute the $n = 1$ eigenmode for an infinite

waveguide whose cross section matches that of our structure at $x = x_A$, we say

```
nb    = 1; # want first eigenmode
vA    = mp.volume( mp.vec(xA, -YP), mp.vec(xA,+YP) ) # c
modeA = f.get_eigenmode(fcen, mp.X, vA, vA, nb, k0, True
```

The return value of `get_eigenmode` is a data structure containing information on the computed eigenmode; for example, to get the group velocity or propagation vector of the mode you could say

```
vgrp     = get_group_velocity(modeA);
k_vector = get_k(modeA)
```

Alternatively, you can call the `meep::fields::output_mode_fields` routine to export the $\mathbf{E}$ and $\mathbf{H}$ components of the eigenmode (at grid points lying in the cross-sectional plane) to an HDF5 file, i.e.

```
f.output_mode_fields(modeA, fluxA, vA, "modeA");
```

where `fluxA` is a `meep::dft_flux` structure created for the cross section described by `vA` . This will create a file called `modeA.h5` containing values of field components at grid points in `vA` .

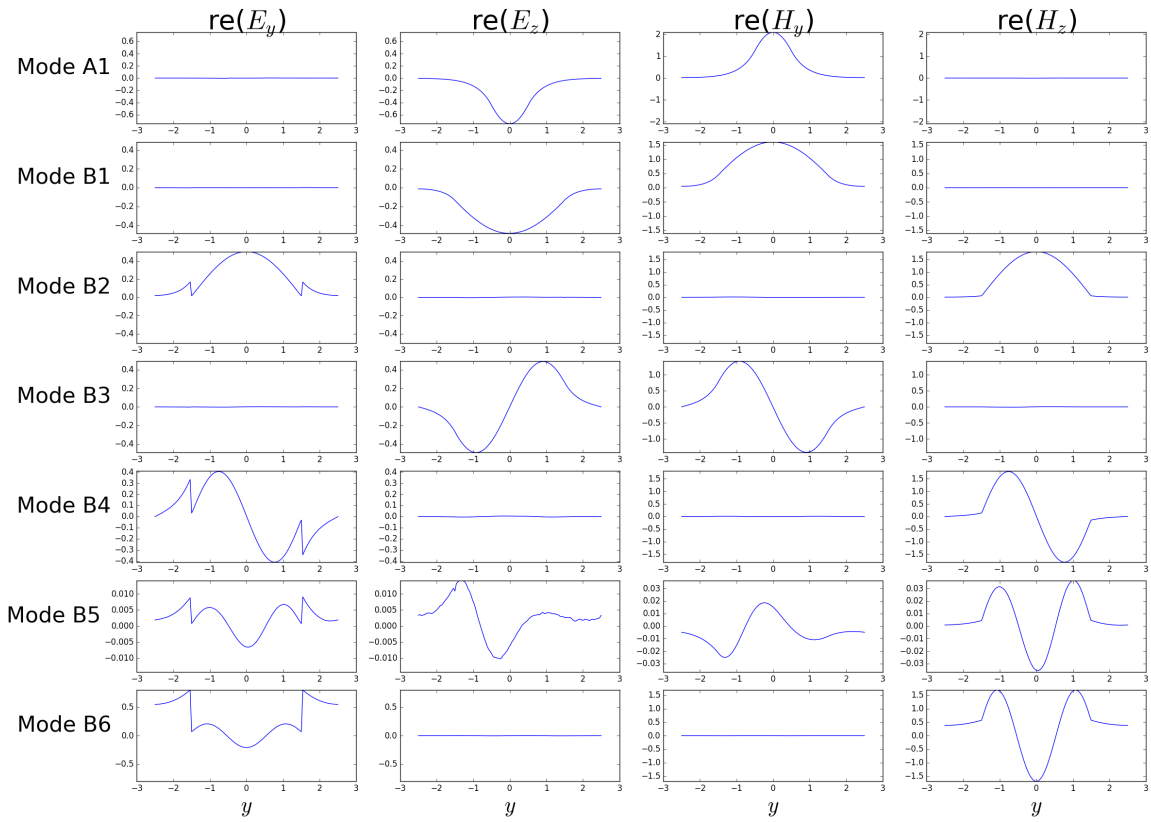
- Having computed eigenmodes with `get_eigenmode` and written their field profiles to HDF5 files with `output_mode_fields` , we can read the data back in for postprocessing, such as (for example) plotting eigenmode field profiles. This is done by the `plot_fields` routine in `wvg-taper.py` ; the basic flow looks something like this:

```
# open HDF5 file
file = h5py.File("modeA.h5", 'r')

# read array of Ey values on grid points
ey = file["ey" + suffix + ".r"][()] + 1j*file["ey" +

# plot real part
plt.plot(np.real(Ey))
```

The `plot_modes` routine in `wvg-taper.py` repeats this process for the lowest mode x_A (`ModeA1`) and the first several modes at x_B (`ModeB1...B6`) and plots the results:



Adding an eigenmode source and timestepping

The next step is to add an *eigenmode source* inside the smaller waveguide---that is, a collection of Meep point sources, lying on the cross-sectional surface at x_A, whose radiated fields reproduce the fields of a given waveguide eigenmode at a given frequency:

```
sources = [ mp.EigenModeSource(src=mp.GaussianSource(fce
                                center=mp.Vector3(xA,0.0)
                                size=mp.Vector3(0.0,LY),
                                eig_band=band_num
                               )
          ]
self.sim=mp.Simulation( cell_size=mp.Vector3(LX, LY),
                        resolution=resolution,
                        boundary_layers=[mp.PML(DPML)],
                        force_complex_fields=True,
                        epsilon_func = eps_func,
                        sources=sources
                      )
```

Next, we timestep to accumulate Fourier-domain fields on a cross-sectional plane within the larger waveguide. This is done by the `get_flux()` method in `wvg_taper.py.

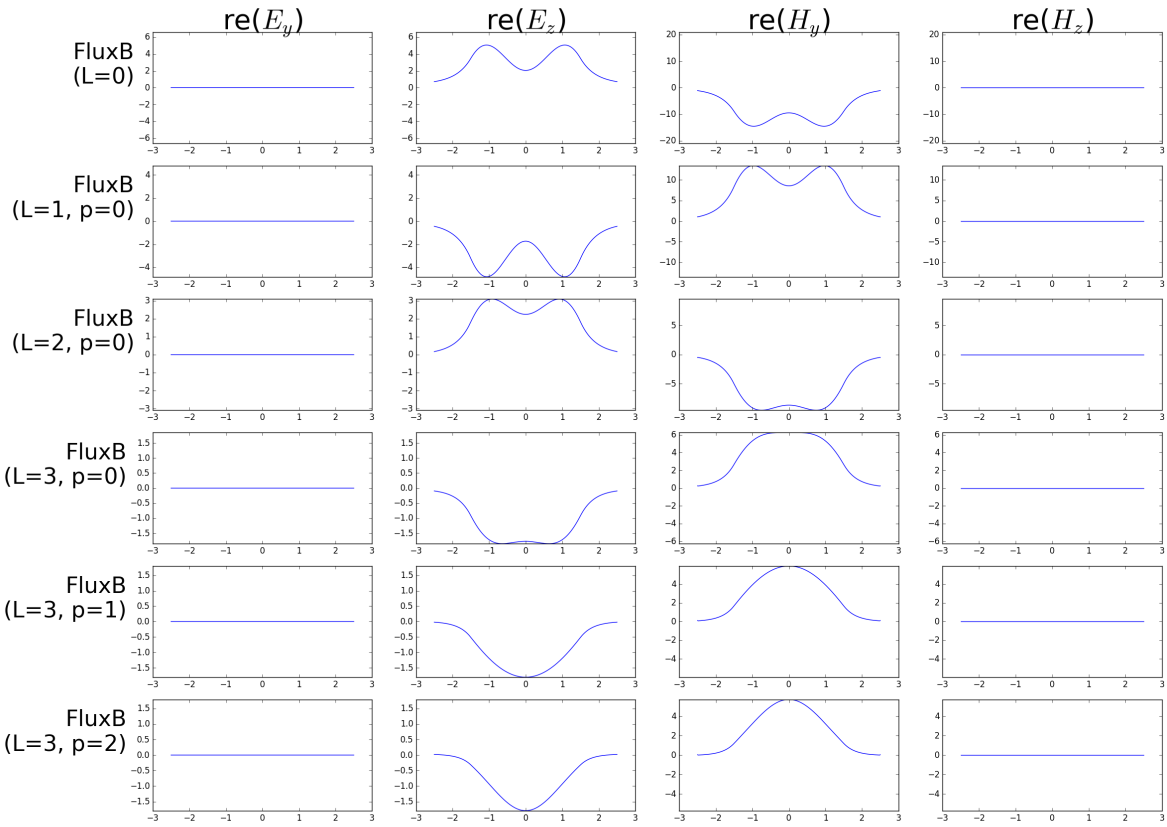
The timestepping continues until the instantaneous Poynting flux through the flux plane at x_B has decayed to 0.1% of its maximum value. When the timestepping is finished, the Fourier-domain fields on the plane at x_B are stored in a `dft_flux` object called `fluxB`. and we can call `meep::fields::output_flux_fields` to export the fields to an HDF5 file, similar to `output_mode_fields` which we used above:

```
f.output_flux_fields(fluxB, vB, 'fluxB')
```

This produces a file called `fluxB.h5` . One slight difference from `output_mode_fields` is that `dft_flux` objects typically store field data for multiple frequencies, so the field-component datasets in the `HDF5` file have names like `ey_0.r` , `ey_1.i` .

Visualizing DFT fields

Having written Fourier-transform fields to HDF5 files, we can read in the data and plot, as we did previously for mode profiles. In the `wvg_taper.py` code this is again handled by the `plot_fields` routine. Here are the results of this process for a few different values of the taper length L and smoothness index p:



Take-home messages:

- For $L = 0$ (no taper, i.e. abrupt junction) the fields at x_B look nothing like the fields of the lowest eigenmode for the larger structure (second row of this plot); clearly there is significant contamination from higher modes.
- As we increase the taper length and the smoothness index the fields at x_B more and more closely resemble the lowest eigenmode fields, indicating that the taper is working to transfer power adiabatically from lowest mode to lowest mode.

Making movies

The `get_flux()` routine in the `wvg_taper.py` supports a keyword parameter `frame_interval` which, if nonzero, defines an interval (in meep time) at which images of the instantaneous Poynting flux over the entire geometry are to be written to `.h5` files. The default is `frame_interval=0`, in which case these images will not be written.

If you specify (say) `frame_interval=1` to `get_flux()` for a geometry with (say) taper length $L = 1.2$ and smoothness index $p = 1$, you will get approximately 100 files with names like

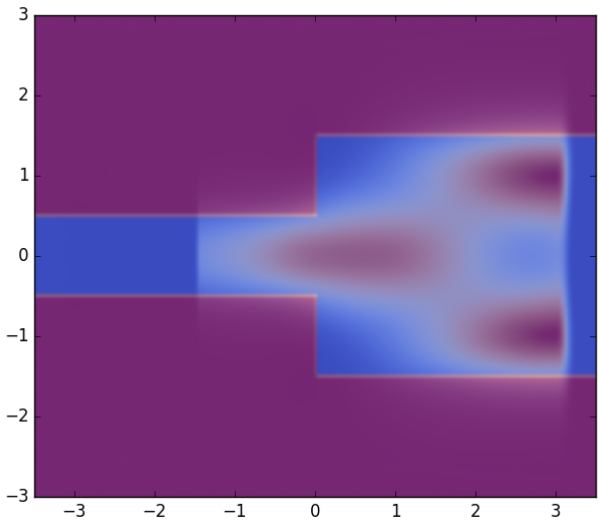
```
L1.2_p1_f1.png
L1.2_p1_f2.png
L1.2_p1_f3.png
...
L1.2_p1_f105.png
...
```

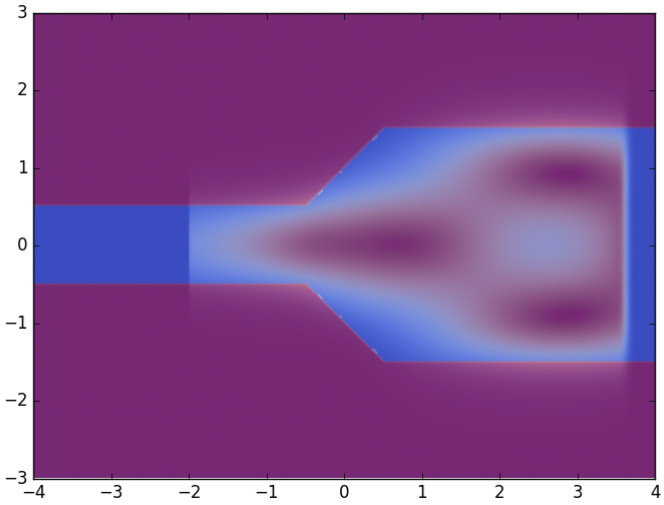
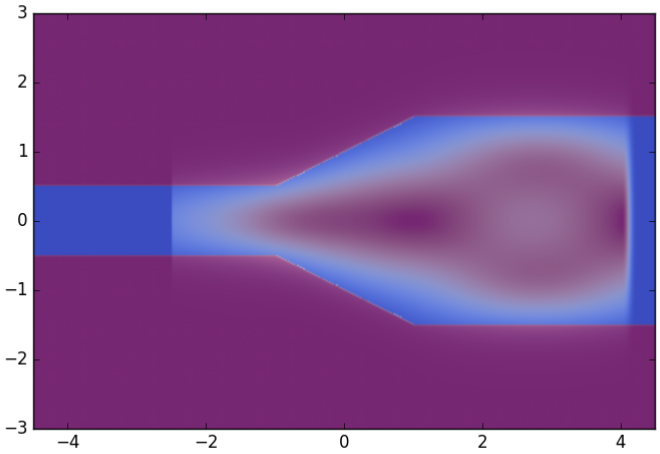
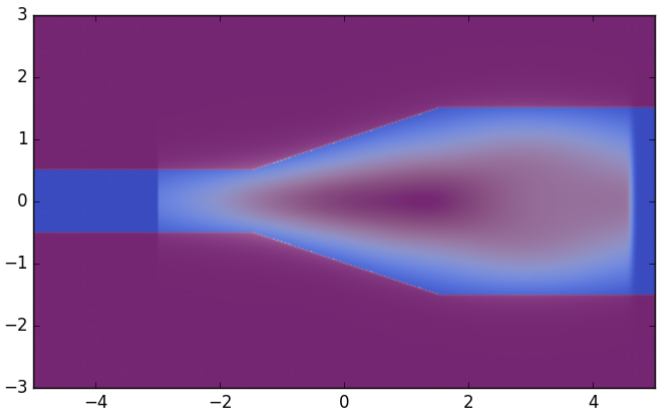
To assemble all these frame files into a movie using FFMPEG, go like this:

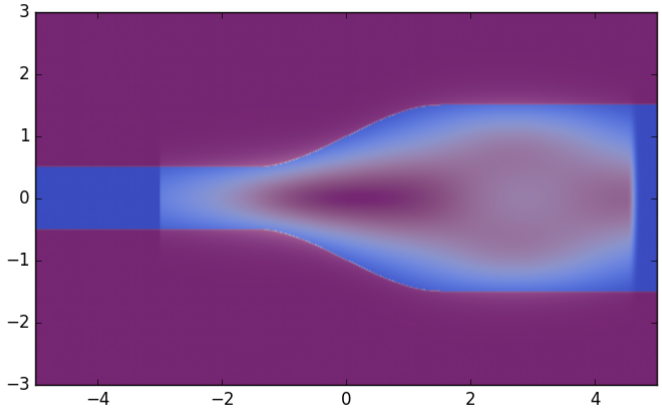
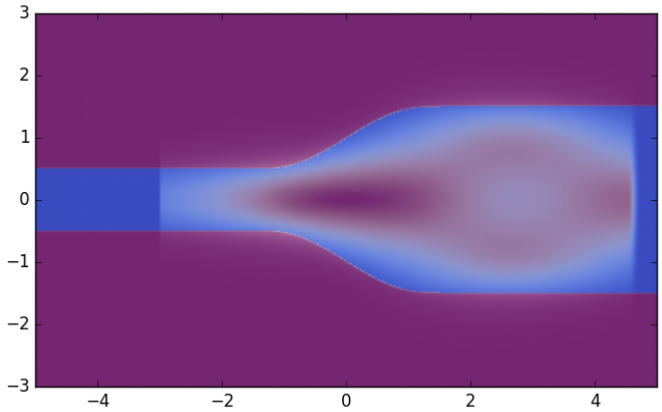
```
# ffmpeg -i 'L1.2_p1_f%d.png' L1.2_p1.mpg
```

(Note that the string `%d` in the input filename is a wildcard that will match all integer values; it needs to be in single quotes to protect it from shell expansion.)

Here are the movies for the various cases considered above:

Taper	*Movie*
L=0	

L=1, p=0	
L=2, p=0	
L=3, p=0	

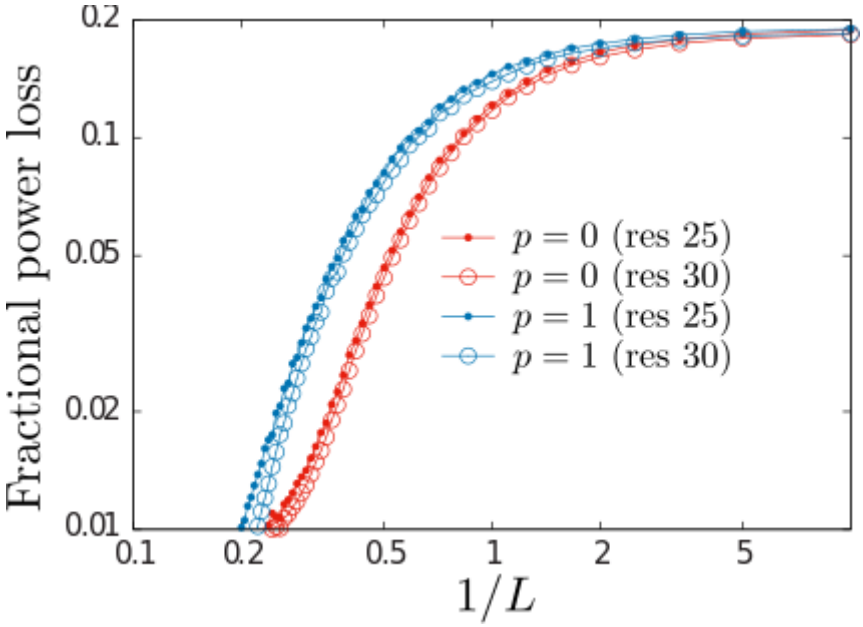
L=3, p=1	
L=3, p=2	

Extracting mode-expansion coefficients

Finally, we call `get_mode_coefficients` to compute the inner product of the Meep DFT fields in the larger waveguide with each of a user-specified list of eigenmodes of the larger waveguide to compute the fraction of the power carried by each mode.

Intra-modal scattering losses vs. taper length and smoothness

Repeating this calculation for many taper lengths L and smoothness indices $p = 0, 1$ yields the following plots showing the rate of decay of inter-mode scattering losses as the taper length $L \to \infty$.



Related computational routines

Besides `get_eigenmode_coefficients`, there are a few computational routines in `libmeep` that you may find useful for problems like those considered above.

Routine for computing MPB eigenmodes (in `mpb.cpp`)

```
void *fields::get_eigenmode(double &omega,
                            direction d, const volume
                            const volume &eig_vol,
                            int band_num,
                            const vec &kpoint, bool ma
                            int parity,
                            double resolution,
                            double eigensolver_tol);
```

Calls MPB to compute the `band_num` th eigenmode at frequency `omega` for the portion of your geometry lying in `where` (typically a cross-sectional slice of a waveguide). `kpoint` is an initial starting guess for what the propagation vector of the waveguide mode will be.

Routines for working with MPB eigenmodes (in `mpb.cpp`)

The return value of `get_eigenmode` is an opaque pointer to a data structure storing information about the computed eigenmode, which may be passed to the following routines:

```
// get a single component of the eigenmode field at a gi
std::complex<double> eigenmode_amplitude(const vec &p, v

// get the group velocity of the eigenmode
double get_group_velocity(void *vedata);

// free all memory associated with the eigenmode
void destroy_eigenmode_data(void *vedata);
```

Routines for exporting frequency-domain fields (in `dft.cpp`)

```
void output_flux_fields(dft_flux *flux, const volume w
                        const char *HDF5FileName);

void output_mode_fields(void *mode_data, dft_flux *flu
                        const volume where,
                        const char *HDF5FileName);
```

`output_flux_fields` exports the components of the (frequency-domain) fields stored in `flux` to an HDF5 file with the given file name. `where` is the `volume` passed to the `flux` constructor. In general, `flux` will store data for fields at multiple frequencies, each of which will

`output_mode_fields` is similar, but instead exports the components of the eigenmode described by `mode_data` (which should be the return value of a call to `get_eigenmode`).

Routines for computing overlap integrals (in `dft.cpp`)

```
std::complex<double> get_mode_flux_overlap(void *mode_
                                           dft_flux *f
                                           int num_fre
                                           const volum

std::complex<double> get_mode_mode_overlap(void *mode1
                                           void *mode2
                                           dft_flux *f
                                           const volum
```

`get_mode_flux_overlap` computes the overlap integral (defined by equation (*) above) between the eigenmode described by `mode_data` and the fields stored in `flux` (for the `num_freq` th stored frequency, where `num_freq` ranges from 0 to `flux->Nfreq-1`.) `mode_data` should be the return value of a previous call to `get_eigenmode`.

`get_mode_mode_overlap` is similar, but computes the overlap integral between two eigenmodes. (`mode1_data` and `mode2_data` may be identical, in which case you get the inner product of the mode with itself; by the normalization convention used in MPB, this should equal the group velocity of the mode.)

Under the hood: How mode expansion works

The theoretical basis of the mode-expansion algorithm is the orthogonality relation satisfied by the normal modes:

$$\langle \mathbf{E}_m^\sigma | \, \mathbf{H}_n^\tau \rangle = C_m \delta_{mn} \delta_{\sigma\tau} \qquad \Big(\{\sigma,\tau\} \in \{+,-\} \Big) \tag{4}$$

where the inner product involves an integration over transverse coordinates:

$$\langle \mathbf{f} | \, \mathbf{g} \rangle \equiv \int_S \Big[\mathbf{f}^*(\vec{\rho}) \times \mathbf{g}(\vec{\rho}) \Big] \cdot \hat{\mathbf{n}} \, dA \tag{5}$$

where S is any surface transverse to the direction of propagation and $\hat{\mathbf{n}}$ is the unit normal vector to S (i.e. just $\hat{\mathbf{z}}$ in the case considered above). The normalization constant C_m is a matter of convention, but in MPB it is taken to be the group velocity of the mode, v_m, times the area A_S of the cross-sectional surface S:

$$C_m = v_m A_S. \tag{6}$$

Now consider a Meep calculation in which we have accumulated frequency-domain $\mathbf{E}^{\text{meep}}$ and $\mathbf{H}^{\text{meep}}$ fields on a `dft-flux` object located on a cross-sectional surface S. Invoking the eigenmode expansion (1) and choosing (without loss of generality) the origin of the x axis to be the position of the cross-sectional plane, the tangential components of the frequency-domain Meep fields take the form

$$\mathbf{E}_\parallel^{\text{meep}} = \sum_n (\alpha_n^+ + \alpha_n^-) \mathbf{E}_{n\parallel}^+, \tag{7}$$

$$\mathbf{H}_\parallel^{\text{meep}} = \sum_n (\alpha_n^+ - \alpha_n^-) \mathbf{H}_{n\parallel}^+, \tag{8}$$

where we used the well-known relations between the tangential components of the forward-traveling and backward-traveling field modes:

$$\mathbf{E}^+_{n\|} = +\mathbf{E}^-_{n\|}, \qquad \mathbf{H}^+_{n\|} = -\mathbf{H}^-_{n\|}.$$

Taking the inner product (5) of both sides of equations (7) and (8) with the $\mathbf{H}$ and $\mathbf{E}$ fields of each eigenmode and using equations (4) and (6), we find

$$\langle \mathbf{H}_m | \, \mathbf{E}^{\rm meep} \rangle = +(\alpha^+_n + \alpha^-_n) v_m A_S$$

$$\langle \mathbf{E}_m | \, \mathbf{H}^{\rm meep} \rangle = -(\alpha^+_n - \alpha^+_n) v_m A_S$$

Thus, by evaluating the integrals on the LHS of these equations---numerically, using the MPB-computed eigenmode fields $\{\mathbf{E},\mathbf{H}\}_m$ and the Meep-computed fields $\{\mathbf{E},\mathbf{H}\}^{\rm meep}$ as tabulated on the computational grid---and combining the results appropriately, we can extract the coefficients $\{\alpha^\pm_m\}$ in the expansion (1). This calculation is carried out by the routine `meep::fields::get_mode_flux_overlap`. (Although simple in principle, the implementation is complicated by the fact that, in multi-processor calculations, the Meep fields needed to evaluate the integrals are generally not all present on any one processor, but are instead distributed over multiple processors, requiring some interprocess communication to evaluate the full integral.)

The Poynting flux carried by the Meep fields (7,8) may be expressed in the form

$$S_x = \frac{1}{2}{\rm Re}\ \langle \mathbf{E}^{\rm meep} | \, \mathbf{H}^{\rm meep} \rangle = \frac{1}{2} \sum_n \left\{ |\alpha^+_n|^2 - |\alpha^-_n|^2) \right\} v_n A_S$$

and thus the fractional power carried by any one (forward- or backward-traveling) eigenmode is given by

$$\text{fractional power carried by } \pm\text{-traveling mode } n = \frac{|\alpha^\pm_n|^2 v_n A_S}{2 S_x}$$

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1. The theory of waveguide modes is covered in many references; one that we have found useful is Snyder and Love, *Optical Waveguide Theory* (Springer, 1983). ↩